A critical analysis of the conceptual and mathematical properties of “instructional efficiency”

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Cognitive load theory (Sweller, 1988) is an influential model in the theory of educational design, despite the fact that it is not without criticism (Schnitz & Kürschner, 2007; De Jong, 2010). One concept that is often used in the analysis of studies that involve measurement of cognitive load is “instructional efficiency”. This was originally defined by Paas and van Merriënboer (1993). Instructional efficiency is introduced as a concept that can provide additional evidence in assessing test performance. A relatively large number of studies, for instance those reviewed by van Gog and Paas (2008), as well as more recent studies (e.g. Künsting, Wirth, & Paas, 2011; Scharfenberg & Bogner, 2010) use the concept of efficiency in the analysis of their results. In most cases, efficiency is used as a supplemental argument in assessing the effects of different modes of instruction (e.g. Kester, Kirschner, & van Merriënboer, 2006; Moreno & Valdez, 2005).

The point of the original concept of instructional efficiency is that effects of instruction can be manifested in different forms. Learners can display better performance on tasks with the same effort invested or achieve the same performance with less effort. In both cases one would be able to say that instruction has resulted in a higher quality of knowledge. The efficiency construct as presented by Paas and van Merriënboer (1993) is a way of combining measured values for mental effort and task performance into a single quantity that is meant to provide a more detailed interpretation of the effects of instruction than task or test performance alone.

In this text, the conceptual and mathematical definitions of efficiency as presented by Paas and colleagues will be critically analyzed. Two aspects of the definition are addressed: mental effort as representing the cost aspect of efficiency and the mathematical form of the definition of efficiency. These analyses will show the conceptual and mathematical inadequacy of this efficiency measure. We further discuss the desired properties of possible alternatives for measuring efficiency.

Mental effort as measure for the cost of performance

Any intuitive definition of “efficiency” should involve weighing product against cost. For instance, when assessing the fuel efficiency of a car one weighs distance covered (the product) against the amount of petrol used (the cost). In the case of instructional efficiency as originally defined by Paas and van Merriënboer (1993), the product is represented by the score on a test and cost is defined to be the mental effort needed to achieve that score. Paas and van Merriënboer targeted mental effort during test performance. Later authors, as pointed out by van Gog and Paas (2008), often used mental effort during instruction, which results in a different measure, weighing the efficiency of the instructional process rather than that of using the resulting knowledge. In both cases the same construct of mental effort is used. The discussion in this article applies to both efficiency constructs. In this it is investigated whether the definition of mental effort serves as a proper representative of the cost invested to achieve the product.
The concept of mental effort is central in cognitive load theory (Sweller, 1988, 1994). The basic assumption of this theory is the notion that human working memory is limited, so that learning, and in particular the acquisition of cognitive schemata, can be inhibited by overload of this limited resource. Cognitive load is considered to be the amount of items occupying a human’s working memory (Sweller, 1988). By acquiring proper cognitive schemata, working memory can be used more efficiently and hence the load should decrease during task performance.

Mental effort is related to cognitive load, but it is not the same. The definition in the original article on efficiency reads: “the total amount of controlled cognitive processing in which a subject is engaged” (Paas & van Merriënboer, 1993). The amount of processing on a certain task should have two dimensions: the intensity of the processing (i.e. the cognitive load) and the duration of the processing. Clearly, mental effort is different from the load per se. However, elsewhere, in an article co-authored by Paas, mental effort is defined as: “the aspect of cognitive load that refers to the cognitive capacity that is actually allocated to accommodate the demands imposed by the task; thus, it can be considered to reflect the actual cognitive load” (Paas, Tuovinen, Tabbers, & van Gerven, 2003, p. 64). Here the two concepts are equated.

In most studies that follow this paradigm, mental effort is measured using a nine-point Likert scale on which subjects indicate their mental effort, ranging from “very very low” to “very very high” (Paas, van Merriënboer, & Adam, 1994). Subjects fill in this scale after completing (parts of) the test. Other researchers take different terms as anchors of the Likert scale, such as “difficulty” instead of mental effort (Kablan & Erden, 2008; Marcus, Cooper, & Sweller, 1996; Yeung, Jin, & Sweller, 1997). While performance can be readily measured as the performance on a test or a benchmark task, the status of mental effort as a measurable quantity is less clear. The operational definition that follows from the way it is measured neglects the duration aspect that should be included in a quantity representing a cost.

The mathematical form of the definition of instructional efficiency

Paas and van Merriënboer (1993) define their efficiency concept by means of the comparison of two or more samples of students. Performance scores (usually a numerical test score) and mental effort scores, using the 9 point Likert scale during test performance, are obtained for all students in each sample. Mental effort may be measured repeatedly during task performance, for instance, after completing each test item, in which case the average mental effort during test performance is used. The grand means and standard deviations of performance and mental effort are computed for the complete sample. Using these statistics, z-scores are computed for mental effort (R) and performance (P) for all subjects in the sample. These z-scores are plotted in a (z(R), z(P))-graph as shown in Figure 1, which is drawn after a similar drawing found in Paas and van Merriënboer (1993). Efficiency is then computed for each subject by computing the perpendicular distance from the point (z(R), z(P)) representing this subject to the line for which z(P) = z(R), as depicted in Figure 1. This results in the following equation for efficiency:

\[ E = \frac{z(P) - z(R)}{\sqrt{2}} \]

where \( P \) stands for Performance score and \( R \) for the Mental Effort score. \( E \) is defined as being positive for the half-plane above the \( z(P) = z(R) \)-line and negative for the other half-plane. Evidently, for the line where \( z(P) = z(R) \), \( E \) vanishes. Paas and van Merriënboer (1993) argue that the position of a point in the \((z(R), z(P))\)-space relative to the \( E = 0 \) line is a representation of “…
the observed relation between mental effort and performance in a particular condition relative to a hypothetical baseline condition, in which each unit of invested mental effort equals one unit of performance (p. 739)."

Figure 1 Graphical representation of the definition of efficiency (E) of the two conditions C1 and C2 as the perpendicular distance from the point \((z(P), z(R))\) to the line for which \(z(P) = z(R)\). After Paas and van Merriënboer (1993).

This definition assumes that mental effort is a resource learners can invest or spend in lesser or greater amounts, and that the amount of effort that is invested is under the control of the learner. This assumption is repeated in Paas and van Merriënboer (1994), who state that “… by investing more mental effort (e.g. trying harder) subjects may reach (…) higher performance …” (p. 355).

The quantitative definition of efficiency also assumes a linear relation between invested effort and performance. In their original article Paas and van Merriënboer (1993) indeed state that efficiency can only be a rough estimate and that performance must approach an asymptote as effort increases.

The interpretation of the quantitative definition of efficiency is not easy. The baseline condition where \(E = 0\) is said to be the condition where one unit of effort “is equal to” one unit of performance. However, in stating this it must be clear what units are used and what equality
means in this case. The origin in the graph in Figure 1 is defined by the mean performance and mean mental effort of the full sample, whereas the scales of the axes are defined by the standard deviations of performance and mental effort. This makes the scales of the graph, and hence any line drawn within the graph, dependent on the complete data set. If, for instance, another condition is considered in the comparison, changing these means and possibly the pooled standard deviations, then the origin of the plane as well as the axis scales will shift accordingly.

The interpretation of the (hypothetical) baseline appears to be that here one extra unit of mental effort will yield one extra unit of performance. Moreover, the original definition states that lines parallel to the $E = 0$ line represent points of equal efficiency that is higher (for lines that are above the $E = 0$ line) or lower than 0. However, for each line parallel to $E = 0$ it is true that one extra amount of effort yields one extra amount of performance. The only difference between parallel lines is the offset, which is the performance at ‘neutral’ or ‘average’ mental effort. This seems at odds with an understanding of efficiency as a weight of product against cost, where a better quality of knowledge schemata should lead to one extra unit of effort yielding more extra performance than for a less efficient performer. The line representing a higher efficiency would thus have a greater slope than the reference $E = 0$ line. Similarly, a line indicating lower efficiency would have a smaller slope. This also means that there is no fixed distance between two efficiency lines. Therefore, the distance from a point on that line to the $E = 0$ baseline is not suitable as a measure because it is not constant along the line. For a given point, the distance to the $E = 0$ line does not have a conceptual interpretation in terms of efficiency.

Efficiency as an additive measure

Central in the computational definition of efficiency is the perpendicular distance from the diagonal line in the $(z(P), z(R))$-space, which can be computed as $(z(P) - z(R)) / \sqrt{2}$. The important step taken here is that a graph is interpreted in terms of a geometric plane described by Cartesian coordinates. The metric of this Cartesian plane depends on the distribution of $P$ and $R$, as the scales of the axes are defined by the standard deviations of these two variables. This means that if these standard deviations are different, the units along the axes are different as well. However, the interpretation of a graph as a plane is mathematically not sound; in a graph, the $x$ and $y$ axes usually have different dimensions, while a basic property of a plane is that both axes can be interpreted as length. Making the axes dimensionless by using $z$-scores does not turn the graph into a plane. Although $z$-scores are dimensionless numbers, they still represent measurements of real world quantities. A difference in $z(P)$, the vertical axis, means a difference in performance. A distance along the horizontal axis means a difference in mental effort, as measured with the cognitive load scale. In Equation (1) effort scores are subtracted from test scores, which is a meaningless operation. Subtracting is an additive operation that can only be applied to quantities that have the same unit. Although $z$-scores are dimensionless numbers they still represent observable quantities with different dimensions. Adding or subtracting two $z$-scores would be the same as computing the fuel efficiency of a car by subtracting the average fuel consumption of a car’s trips from the average distance traveled by the car on these trips. Obviously liters cannot be subtracted from kilometers; it is not possible to solve this by computing the $z$-scores for fuel consumption and distance. This would yield a computation such as $(D / sd(D) - F / sd(F))$ (with $D$ for distance and $F$ for fuel consumption), which would a number which has no clear interpretation.
The behavior of the efficiency measure

Computational problems arise as a direct consequence of the flawed mathematical definition of efficiency. Because of the use of $z$-scores, the measure for efficiency becomes dependent on the standard deviations of both the mental effort and performance scores in the sample. This results in the situation that efficiency can be different for equal values of effort and performance. A simulated example can make this clear. Suppose that we are comparing two equal-sized groups with means $\bar{P}_1 = 50, \bar{R}_1 = 5.0$ for the first group and means $\bar{P}_2 = 60, \bar{R}_2 = 6.0$ for the second. For the pooled sample this results in grand means $\bar{P} = 55, \bar{R} = 5.5$. In order to compute efficiencies, we also need to know the standard deviations of the complete sample. Assume the standard deviation for $P$, $\text{sd}(P) = 10$ (for the pooled sample), and $\text{sd}(R) = 1.0$. In this case the efficiencies $E_1 = E_2 = 0$. But, if the distributions of $P$ or $R$ are different, the picture may be different for the same values for $\bar{P}_1, \bar{P}_2, \bar{R}_1$ and $\bar{R}_2$. Suppose that in the above example the total standard deviation for $P$ is not 10 but 7, meaning a sharper distribution of the performance scores, but the means for $P$ and $R$ as well as the distribution of $R$ remain the same. Efficiencies can now be computed and yield $E_1 = -0.15$ and $E_2 = 0.15$. If the distribution of $P$ is instead somewhat broader, with a standard deviation of 20, the efficiencies become $E_1 = 0.18$ and $E_2 = -0.18$. Note that in this case $E_1 = -E_2$ always, as a consequence of the definition of $z$-scores and having equal-sized groups.

Varying standard deviations in $P$ means that the $E$s not only have a different size, but their sign is also different. In the first case $E_1 = E_2 = 0$, in the second case $E_1 < E_2$ and in the third case $E_1 > E_2$. So for a different standard deviation but with the same mean values for $P$ and $R$ the decision as to which group is the most efficient can be different.

The reason for this behavior is that the units along the axes in the $(z(P), z(R))$-space are taken as the standard deviations in $P$ and $R$, and thus will vary as a function of the spread of the data points. If the spread in $P$ is relatively larger than the spread in $R$, $P$ will contribute less to the efficiency number than $R$. So, more weight is given to either performance or mental effort depending on the distribution in the sample. Note that this is completely independent of the sources of the variance in either $P$ or $R$, which may include heterogeneity of the population or noise in either the performance or the mental effort measures. According to Paas and van Merriënboer’s definition, with the same mean values for performance and mental effort, in some cases condition 1 can be called more efficient than condition 2, but condition 2 can become more efficient than condition 1 by changing only the spread of values.

Rethinking the efficiency concept

In the above sections it becomes clear that the mathematical form of the efficiency concept is flawed, because it has problems of interpretation and because it generates contradictory outcomes. Therefore, the question is whether the concept of efficiency can be defined in an alternative way that captures the intuitive notion put forward by Paas and van Merriënboer (1993). Some issues to take into account when considering such alternatives are offered in this section. However this does not result in the proposal of a specific alternative measure, as situations may vary in what is considered to be ‘efficient’ knowledge, as well as in what properties of performance and effort can be measured.

In rethinking the concept of efficiency, we should reconsider the intent of the concept and whether we need it at all. According to the original authors, the efficiency concept is meant to
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indicate the quality of the cognitive schemata acquired during the learning process. This is exemplified by the following quote:

“Mental effort in combination with performance measures will provide us with a better, more subtle indicator of the quality of learning outcomes, that is, in terms of the efficiency of cognitive schemata acquired, elaborated, or automated as a result of instruction, and hence with a better indicator of the quality of different instructional conditions.” (Van Gog & Paas, 2008, p. 20)

The higher the quality of cognitive schemata acquired during instruction, the more efficiently they can be applied in testing tasks. Therefore, it seems appropriate to seek further than alternative combinations of combining performance and mental effort. This broader quest takes us beyond the cognitive load literature and is basically a quest for high quality assessment. In this section, we start the discussion with the original concepts of load and effort, and expand it to alternative means for assessing the quality of cognitive structures resulting from instruction.

One obvious alternative to the efficiency measure would keep the original $P$ and $R$ and simply take their quotient: $E = P / R$. This has actually been done by Kalyuga and Sweller (2005), who then take as a reference value $P_{\text{max}} / R_{\text{max}}$, the maximum scores for performance and effort. The reason they give is that they needed a real time estimate of efficiency, during task performance, which is impossible for a score based on z-scores. But it is clear from the above that this measure would always be preferable to that of Paas and van Merriënboer (1993). As in many cases where some kind of cost is weighed against the product, such as the distance (in km) you can travel on a liter of gasoline in the case of fuel efficiency, the natural path is to divide, not to subtract. Even if the scales are non-linear, the quotient can still be meaningful. Efficiency need not be constant along the effort axis. For instance if performance approaches an asymptote, the efficiency will probably decrease. This has the interpretation that it becomes more and more costly to improve an already high performance. In many cases this measure could give a reasonable indicator that does not have the problems described in this paper: no z-scores with changing scales and no unit conflicts. Moreover, it defines efficiency in an absolute sense rather than in a relative sense as seen in Paas and van Merriënboer’s definition. The efficiency in Paas and van Merriënboer’s definition depends not only on the performance and load of a subject or condition, but also on the values obtained for all other subjects including those in other conditions. An absolute measure makes the efficiency measure independent of the particular experimental design, and adds utility for doing more than comparing instructional conditions, such as investigating the influence of psychometric factors such as intelligence or test anxiety.

In the work using efficiency mental effort was used as the indicator for the cost dimension of efficiency. In the introduction it was mentioned that Paas and colleagues fail to distinguish between cognitive load and mental effort as measurable quantities, and that mental effort measured using the 9-point Likert scale does not have the properties of a cost. A true cost measure should be the product of intensity and time, totaling to a total amount of effort spent.

Alternatives for measuring mental effort exist such as the NASA-TLX (Hart & Staveland, 1988). Schnotz and Kürschner (2007) discuss a number of alternatives for the measurement of cognitive load, including several physiological measures. A problematic aspect is that these measures all address “cognitive load” as an undifferentiated concept, whereas cognitive load theory distinguishes “intrinsic”, “extraneous”, and “germane” cognitive load (Sweller, van Merriënboer, & Paas, 1998). These different kinds of load were introduced to differentiate between cognitive load that is implicit in the task itself load that is a consequence of task irrelevant aspects of the
environment and load that is devoted to the construction of new cognitive schemata. This article is not the place to discuss these cognitive load issues, but, without going into the details, one could argue that at the least, “extraneous load” should be separated from the load that is associated with the task itself when considering efficiency as a measure of the quality of cognitive schemata.

In choosing a measure for the effort spent on a task the amount of time used for the task should be seriously considered. Time may even be considered as the single measure for effort. This has two main advantages. First, it clearly has the linear properties needed in computation. Spending twice as much time is clearly doubling the cost, which cannot easily be said for a Likert scale. Second, the amount of time spent (when not constrained in the testing conditions) is clearly under the control of the learner. Of course also the intensity of task performance during the time spent should be measured. The cost should be a product of (average) intensity and time. Time on task has been used by a number of researchers and has even been suggested as a measure of mental effort by van Gog and Paas (2008). By using only time, the assumption is that the average intensity does not vary significantly between learners. In specific testing situations, this assumption should be validated.

Conclusion

In this article it is argued that the efficiency measure as introduced by Paas and van Merriënboer (1993) has some serious conceptual and computational limitations. The main argument is that the mathematical form of the efficiency measure is flawed. By subtracting $z$-scores of quantities with different dimensions, one cannot arrive at a meaningful number. The computational examples make it clear that contradictory results may indeed arise and that Paas and van Merriënboer’s efficiency cannot be considered to be even the rough estimate of the efficiency of knowledge as Paas and van Merriënboer (1993) argue. Conclusions based on this measure or on its variants found in many studies as listed by van Gog and Paas (2008) should be reconsidered.

At the conceptual level, the issue of mental effort as representing the cost dimension of efficiency was raised. Paas and colleagues make no distinction in the operationalization of “mental effort” and “cognitive load”, nor do they measure the differences between the different kinds of cognitive load (intrinsic, extraneous, and germane) that they describe in their theoretical model. This makes it hard to interpret the load measure as an indicator of the “mental” cost that subjects invest in performing the test task. On this point the discussion may continue. Using objective measures of processing capacity and taking into account the period of time over which this capacity is used may yield adequate quantitative measures of effort.

As efficiency is meant to be an indicator of the quality of cognitive schemata, how we measure this quality may depend on the kind of tasks involved. But, if a measure involves a similar performance vs. load balancing, it would be better to base the model on a quotient $(P / R)$ rather than a difference $(z(P) - z(R))$. Apart from avoiding the mathematical issues with Paas and van Merriënboer’s instructional efficiency, these alternatives yield absolute rather than relative values for efficiency that have the advantages that (1) they are independent of a particular experimental design and (2) they allow for the investigation of the effects on efficiency of factors other than instructional condition. Interestingly, the relativity of efficiency in Paas and van Merriënboer’s definition is not always recognized, which can lead to incorrect interpretation of results. For instance, Yeung, Jin and Sweller (1997) compare efficiencies based on two different performance measures (vocabulary and comprehension) having different mean scores and standard deviations.

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The resulting comparison of these efficiency scores is meaningless, as they are effectively computed for different scales.

Given the fact that time on task can be measured more accurately than mental effort, and that time of task can also be controlled, time would in many cases be a better measure of cost as part of the efficiency construct, because it will be much easier to compare conditions simply by controlling either time or performance. Of course, a potential drawback would be that there is no guarantee that the time measured is really dedicated to performance on the test. In controlled situations this may be avoided by stressing the importance of speed in the testing situation.

In many cases it may not even be necessary to measure mental effort or time on task separately. If it is possible to design a performance measure that directly reflects the quality of knowledge, a separate efficiency construct may not be needed. Taken to its essence, efficiency is equivalent to the quality of the knowledge gained in instruction. Therefore, the idea of having an efficiency concept separate from performance may be superfluous. In realistic situations the quality of the knowledge as defined in the specific context for which the training is designed should be what is measured by the performance test. If performance speed or mental effort is considered to be an important aspect of the performance, it should be made part of the assessment itself, that is, be part of the scoring system or be under control in the testing procedure, rather than adding an extra quantity in order to interpret the data, such as efficiency. This would mean designing test situations where either time on task (“try to solve as many items in a given time”) or performance (“solve a given set of problems as fast as possible”) is kept constant. Such integrated measures share the property of being absolute rather than relative measures and their score relates directly to the quality of knowledge. Moreover they abolish the need for combining two numbers.

In the discussion we did not address at any length the difference that has been drawn between performance efficiency as opposed to instructional efficiency. As noted in van Gog and Paas (2008), many authors use mental effort during instruction as the cost dimension as opposed to mental effort during test performance (the “original efficiency”). Both kinds of efficiency, as discussed in van Gog and Paas (2008), use the same mathematics, and hence the same arguments apply against them. Again, as a measure of instructional cost, time on task – in this case, time taken for instruction – deserves consideration as an alternative to mental effort. Tuovinen and Paas (2004) introduce a way of combining performance with both training effort and performance effort by replacing the (P,R)-plane by a (P, ET, EP)-space resulting in a formula for “3D-efficiency” as (P - ET - EP) / √3. It goes without saying that this will result in the same mathematical problems. Similar mathematical arguments hold against a measure for “Involvement”, introduced by Paas, Tuovinen, van Merriënboer, and Darabi (2005) and defined as (z(P) + z(R)) / √2. The mathematical form of all these variants of combining effort and performance is flawed, and should be reconsidered.

References


